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AC Line Power Oscillations

Current and Power in the Time and the Frequency Domains

This note treats the problem of whether the low-frequency power oscillations that are sometimes seen in electric AC-AC locomotives can potentially interfere with DC track circuits. The note presents a set of actually recorded line current and voltage signals as raw data (i.e., in the time domain), and as spectra found by means of a FFT analysis. In parallel, sets of expressions that approximate the recorded signals are treated mathematically. The data was recorded on a 16.7 Hz electrified railway.

It is shown that the recorded 1.6 Hz power oscillations are in fact due to amplitude modulation of the line current and the line voltage. This amplitude modulation causes components with the frequencies 16.7 ± 1.6 Hz to be present in the current and voltage, this in turn causing the 1.6 Hz component to be found in the line power.

This means that no low-frequency components are present in the line current. In other words, a low-frequency power oscillation does not in itself cause with interference with DC track circuits.

A good correlation is found between the recorded and the synthetic signals.

Line Current and Voltage

The 2 upper curves of figure 1 (2^{nd} last page) show the line current and line voltage that were actually recorded.

The line current signal shows that the amplitude of the 16.7 Hz fundamental varies between approximately 200 A and 400 A with a frequency of approximately 1.6 Hz. This means that the average fundamental amplitude is approximately 300 A, and the amplitude of the 1.6 Hz modulation signal is approximately 100 A. Mathematically, this is written as

$$i_{line}(t) \quad = \quad \hat{i} \cdot \sin(\omega_F t) \cdot (1 - k_i \cdot \cos(\omega_R t)), \qquad [1]$$

with $\hat{i} \approx 300$ A and $k_i \approx 1/3$ (i. e., 100 A/300 A), and with ω_F and ω_R being the angular frequencies of the 16.7 Hz fundamental and the 1.6 Hz modulation, respectively.

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Thus, in [1], the term $\hat{i} \cdot \sin(\omega_F t)$ expresses the 16.7 Hz fundamental with the average amplitude of ≈ 300 A, while the term $(1 - k_i \cdot \cos(\omega_R t))$ gives the amplitude variation.

The same applies to the line voltage signal, however, the amplitude of the modulation signal is much smaller:

$$u_{line}(t) = \hat{u} \cdot \sin(\omega_F t) \cdot (1 + k_u \cdot \cos(\omega_R t)) \qquad [2]$$

Here, reading the graph gives $\hat{u} \approx 22$ kV and $k_u \approx 1/20$ or 5 %.

It should be noticed in [1] and [2] that no phase angle difference between the current and voltage signals has been considered. No phase angle difference is readable from the curves, not even in a magnified view, and as it will be shown later on when analysing the line power, the phase angle difference, if any, is negligible.

It should also be noticed that the modulation cosines have opposite sign in [1] and [2]. This is due to the fact that the voltage variation is due to the voltage drop across the line – more current gives less voltage at the train. This is also seen in a magnified view.

Multiplying the two terms in each of [1] and [2] gives:

$$i_{line}(t) = \hat{i} \cdot \sin(\omega_F t) - \frac{1}{2}\hat{i} k_i \{\sin((\omega_F + \omega_R)t) + \sin((\omega_F - \omega_R)t)\} \qquad [1a]$$

$$u_{line}(t) = \hat{u} \cdot \sin(\omega_F t) + \frac{1}{2}\hat{u} k_u \{\sin((\omega_F + \omega_R)t) + \sin((\omega_F - \omega_R)t)\} \qquad [2a]$$

I. e., each of [1a] and [2a] consist of 3 terms added together:

- The fundamental with the angular frequency ω_F and an amplitude of $\hat{i}$ (or $\hat{u}$)
- A term with an angular frequency of $\omega_F + \omega_R$ and an amplitude of $\frac{1}{2}\hat{i}k_i$ (or $\frac{1}{2}\hat{u}k_u$)
- A term with an angular frequency of $\omega_F - \omega_R$ and an amplitude of $\frac{1}{2}\hat{i}k_i$ (or $\frac{1}{2}\hat{u}k_u$)

This is also seen in the upper 2 curves of figure 2 (last page), where the said components are easy to identify. These curves are the spectra of the curves in figure 1, i.e., they are calculated from the actually measured signals.
The sidebands of the 16.7 Hz current fundamental are located at 15.1 Hz and 18.3 Hz, respectively, and the amplitudes are approximately 50 A as expected from [1a].

The current spectrum also shows small components at 16.7 ± 0.8 Hz. These are not considered in the following analysis of the synthetic signals.

Line Power

In the time domain, the electrical power is the product of voltage and current:

$$p_{line}(t) \quad = \quad u_{line}(t) \cdot i_{line}(t) \qquad [3]$$

This multiplication has been carried out with the recorded signals, and the result is given in the lower curve in figure 1. The line power consists of a positive average component (i. e., what is called active power in the jω-domain[1]), a 33.3 Hz component, and modulation components.

The fact that $p_{line}(t)$ is always positive (above the zero axis) shows that the current and the voltage fundamentals are in phase. Any phase difference would lead to zero-crossings of the 33.3 Hz component (and to reactive power in the jω-domain).

Going back to the synthetic time domain mathematical expressions, inserting [1a] and [2a] into [3] gives the following (the figures u_n and i_n in the parenthesis refer to the 1st, 2nd, and 3rd term in [2a] and [1a], respectively):

$$p_{line}(t) \quad = \quad \frac{1}{2}\hat{u}\hat{i} - \frac{1}{2}\hat{u}\hat{i} \cdot \cos(2\omega_F t) \qquad (u_1 \cdot i_1\text{, 0 Hz and 33.3Hz})$$

$$-\frac{1}{4}\hat{u}\hat{i}k_i \cdot \cos(-\omega_R t) + \frac{1}{4}\hat{u}\hat{i}k_i \cdot \cos((2\omega_F + \omega_R)t) \qquad (u_1 \cdot i_2\text{, 1.6 Hz and 34.9 Hz})$$

$$-\frac{1}{4}\hat{u}\hat{i}k_i \cdot \cos(\omega_R t) + \frac{1}{4}\hat{u}\hat{i}k_i \cdot \cos((2\omega_F - \omega_R)t) \qquad (u_1 \cdot i_3\text{, 1.6 Hz and 31.7 Hz})$$

$$+\frac{1}{4}\hat{u}\hat{i}k_u \cdot \cos(\omega_R t) - \frac{1}{4}\hat{u}\hat{i}k_u \cdot \cos((2\omega_F + \omega_R)t) \qquad (u_2 \cdot i_1\text{, 1.6 Hz and 34.9 Hz})$$

$$-\frac{1}{8}\hat{u}\hat{i}k_u k_i - \frac{1}{8}\hat{u}\hat{i}k_u k_i \cdot \cos((2\omega_F + 2\omega_R)t) \qquad (u_2 \cdot i_2\text{, 0 Hz and 69.8 Hz})$$

$$-\frac{1}{8}\hat{u}\hat{i}k_u k_i \cdot \cos(2\omega_R t) - \frac{1}{8}\hat{u}\hat{i}k_u k_i \cdot \cos(2\omega_F t) \qquad (u_2 \cdot i_3\text{, 3.2 Hz and 33.3 Hz})$$

$$+\frac{1}{4}\hat{u}\hat{i}k_u \cdot \cos(-\omega_R t) - \frac{1}{4}\hat{u}\hat{i}k_u \cdot \cos((2\omega_F - \omega_R)t) \qquad (u_3 \cdot i_1\text{, 1.6 Hz and 31.7 Hz})$$

$$-\frac{1}{8}\hat{u}\hat{i}k_u k_i \cdot \cos(-2\omega_R t) - \frac{1}{8}\hat{u}\hat{i}k_u k_i \cdot \cos(2\omega_F t) \qquad (u_3 \cdot i_2\text{, 3.2 Hz and 33.3 Hz})$$

$$-\frac{1}{8}\hat{u}\hat{i}k_u k_i - \frac{1}{8}\hat{u}\hat{i}k_u k_i \cdot \cos((2\omega_F - 2\omega_R)t) \qquad (u_3 \cdot i_3\text{, 0 Hz and 63.4 Hz})$$

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[1] The jω-method is a mathematical trick that eases the treatment of stationary sinusoidal signals. The term *active* power used in jω-analysis equals the average power found here, i. e., the "DC component" of the power spectrum seen in figure 2, lower curve. The *reactive* power in the jω-analysis quantifies the amount of power which is actually shifted forth and back between the supply and the load at a rate of 33.3 Hz (at a 16.7 Hz fundamental). Based on values from the spectrum of $p_{line}(t)$ (figure 2, lower spectrum), the reactive power could be calculated as $reactive\ power = \sqrt{(amplitude\ of\ 33.3\,Hz\ component)^2 - (DC\ component)^2}$. If the jω-method is used with signals comprising harmonic components, each harmonic must be treated individually.

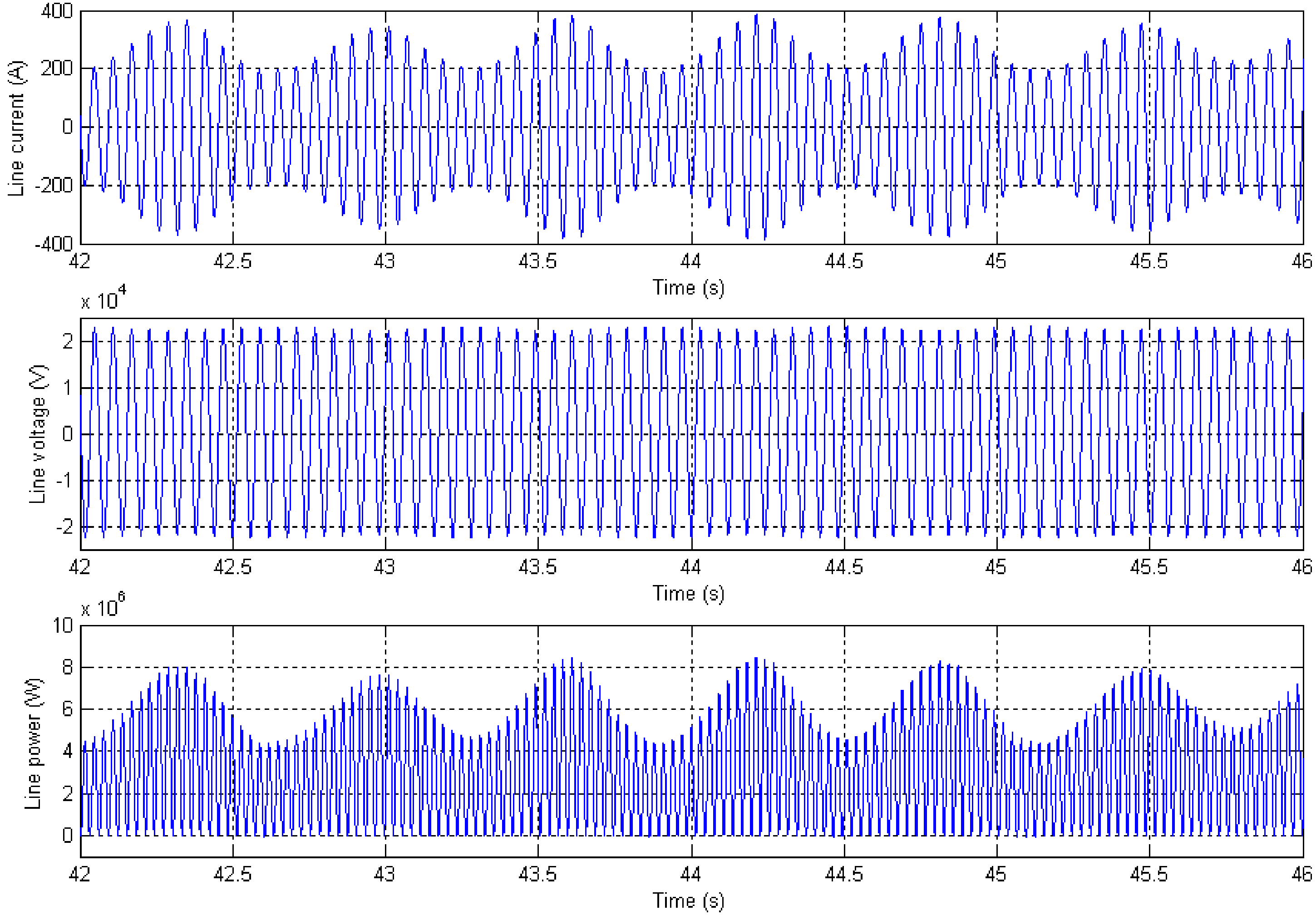
With cos(-x) = cos(x), and with the approximate values previously used, this gives the components of table 1 below:

Frequency component	**Expression**	**Approximate value**
0 Hz (active power)	$\frac{1}{2}\hat{u}\hat{i}\cdot\left(1-\frac{1}{2}k_u k_i\right)$	3.2 MW
1.6 Hz	$\frac{1}{2}\hat{u}\hat{i}\cdot\left(k_i-k_u\right)$	935 kW
3.2 Hz	$\frac{1}{4}\hat{u}\hat{i}k_u k_i$	28 kW
31.7 Hz	$\frac{1}{4}\hat{u}\hat{i}\cdot\left(k_i-k_u\right)$	467 kW
33.3 Hz	$\frac{1}{2}\hat{u}\hat{i}\cdot\left(1-\frac{1}{2}k_u k_i\right)$	3.2 MW
34.9 Hz	$\frac{1}{4}\hat{u}\hat{i}\cdot\left(k_i-k_u\right)$	467 kW
63.4 Hz	$\frac{1}{8}\hat{u}\hat{i}k_u k_i$	14 kW
69.8 Hz	$\frac{1}{8}\hat{u}\hat{i}k_u k_i$	14 kW

Comparing to the components of the line power spectrum (figure 2, lower curve), the values of the table are a little bit too high, but as the upper curve in figure 2 shows, this is due to the fact that the original reading (from figure 1, upper curve) of an average 16.7 Hz current of 300 A is too high. A value of 280 A would be more correct.

The power spectrum shows small components at 0.8 Hz. These are due to the line current components at 16.7 ± 0.8 Hz that were not included in the synthetic signals.

Signals in the time domain



Signals in the frequency domain